Max-Min Fair Resource Allocation for LTE-Advanced Relay-Enhanced Cells

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Abstract—Relaying in OFDMA-based networks is an effective mechanism for enhancing the cell-edge capacity, extending the coverage and utilizing the bandwidth efficiently through spatial reuse. To harness the benefits of relaying, efficient resource allocation schemes should be used, which are aware of the highly dynamic nature of interference in OFDMA relay-enhanced cells (RECs). This paper tackles the joint Power Allocation (PA) and Resource Block Assignment (RBA) problem in a single-cell OFDMA REC. Fair allocation of resources is achieved by assuming a max-min fair objective for the problem and a novel solution technique is proposed, which is capable of obtaining a local optimum for this complex problem. Comparison with other similar works shows the effectiveness of our proposed technique. Simulation results show increase in the 10%-tile capacity by a factor of 6.6 compared to previous solution techniques. Moreover, using Jain’s fairness index, we show that this technique guarantees more fairness among users.

Keywords-component; DF Relaying; Spatial Reuse; Resource Allocation; Max-Min Fair; DC Programming.

I. INTRODUCTION

To meet the ambitious performance goals of International Mobile Telecommunication (IMT)-Advanced in next generation networks, the Third Generation Partnership Project (3GPP) has developed the new mobile communication standard, LTE-A [1]. One of the main problems of LTE-A networks, however, is the aggressive frequency reuse (reaching unity) among adjacent cells, which has become a common trend in next generation networks to achieve higher system capacity. This results in a large inter-cell interference being seen by Cell-Edge (CE) User Equiments (UEs). This problem, along with the already poor links of these users to the eNB, causes small signal-to-interference-plus-noise-ratio (SINR) at the CE receiver, which leads to the small rates achieved by CE UEs.

Multihop communication is a very promising technique to solve the problem of CE UEs, where the poor eNB-UE link is split, through an intermediate relay station (RS), into stronger eNB-RS and RS-UE links, thus mitigating the path loss effect. Moreover, relaying is used to increase the capacity by enabling spatial reuse, where multiple transmissions take place simultaneously in the same frequency/time slot throughout a cell [2]. Spatial reuse is feasible thanks to the small transmit power of RSs and the high path loss in typical urban environments [3]. 3GPP has included Decode-and-Forward (DF) relaying as part of the LTE-A standard, where the RS receives the message from the eNB, fully decodes it, then re-encodes it and re-transmits it to the CE UE. To exploit the full benefits of relaying, however, efficient resource allocation (RA) techniques should be used, which take into account the inter-cell interference and intra-cell interference (due to spatial reuse within the sectors of the cell). This makes the RA problem in OFDMA-based relay-enhanced cells (RECs) quite challenging [4] especially when noting that the placement of relays brings interference closer to the CE UEs of adjacent cells. Moreover, relays increase the inter-cell interference dynamics dramatically, which complicates the interference predictability and raises the need for more intelligent dynamic RA techniques that balance between the aggressive resource reuse and the efficient management of the resulting inter-cell interference.

The RA problem has been extensively studied via different approaches in the literature. The main techniques used include efficient resources allocation of subcarriers (SCs) within OFDMA context as in [5] and [6], efficient power control techniques (to mitigate interference) as in [7], optimal RS assignment (for cooperative RSs) as in [5] and [8]. Most of the current algorithms use one or more of these techniques jointly.

In this paper, we tackle the joint power allocation (PA) and resource block assignment (RBA) RA problem with a max-min fairness objective. We chose this objective because we noted that the majority of previous works in RA problem focused on the sum capacity as an objective function, which is indeed good for maximizing the whole system capacity. However, it is unfair for the CE UEs as it assigns low rates for them because of their bad channel conditions. This is dangerous because if any CE UE falls in outage state, the whole multiuser system will also fall in an outage state [9, Eq. (25)]. Most of the previous works used the dual decomposition technique proposed by Yu et al in their seminal work [10] to solve similar RA problems. However, this technique suffers from its slow rate of convergence [11]. Moreover, solving the dual problem is not efficient for nonconvex problems containing both continuous and binary variables because of the non-zero duality gap of these problems [12]. Accordingly, we propose a novel iterative algorithm for the RA problem based on dividing it into two sub problems; one for PA and the other for RB assignment, then solving each sub problem alone assuming the variable of the other sub problem as fixed. To solve the PA sub problem, we use the Difference of two Convex functions (DC) programming [13] optimization technique, which started to draw attention in the power control literature. Results show the convergence of our technique to an optimal solution.

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The rest of the paper is organized as follows: the system model is presented in Section II. The optimization problem formulation is detailed in Section III and the proposed solution approach is presented in Section IV. Another typical solution approach that will be used for comparison and benchmarking will be presented in Section V. Performance evaluation results are shown and discussed in Section VI before the paper is finally concluded in Section VII.

II. SYSTEM MODEL

We consider a single OFDMA cell (Fig. 1) where $M$ in-band DF RSs are placed on a circle of radius $R_{RS}$ at equal angular distances from each other and $K$ UEs are distributed uniformly in the cell. The sets of indices of RSs and UEs are $M = \{1, 2, \ldots, M\}$ and $K = \{1, 2, \ldots, K\}$, respectively. The users in each cell are classified as either Cell-Center (CC) UEs or CE UEs according a threshold average channel gain $h_{th}$ whose value is determined empirically. The sets of indices of CC UEs and CE UEs are $\mathcal{K}^{CC}$ and $\mathcal{K}^{CE}$, respectively, where $\mathcal{K} = \mathcal{K}^{CC} \cup \mathcal{K}^{CE}$. In the context of LTE-A, the eNB-UE, eNB-RS and RS-UE links are called the macro access links, backhaul links and relay access links, respectively. The relay selection process for CE UEs is determined based on the best received reference signal criterion, which exists in the LTE-A standard [1]. The whole bandwidth is divided into a set of $N$ RBs defined by the set $\mathcal{N} = \{1, 2, \ldots, N\}$. We assume reuse of the RBs in the same cell, where the eNB reuses the RBs assigned for the relay access links to transmit to CC UEs. This form of spatial reuse implies multiple simultaneous transmissions on a certain RB in the same cell, which implies, in turn, the presence of intra-cell interference. For simplicity, we do not take the inter-cell interference into account, i.e., we consider the intra-cell interference as the dominant source of interference.

![Figure 1: System Model](image)

The time resource allocation in our system consists of 2 timeslots (TSs) as shown in Fig. 2. In the 1st TS, the eNB transmits messages to the CC UEs and RSs on orthogonal RBs. In the 2nd TS, the eNB continues its transmission to the CC UEs (on the same RBs of the 1st TS or different RBs) and the RSs decode the received messages and re-transmit them to the respective users on the same RBs of the 1st TS. To increase the cell capacity, the eNB can reuse some RBs, which are assigned to RS-UE links, to transmit messages to the CC UEs as shown in Fig. 2.

![Figure 2: Time Resource Allocation](image)

III. OPTIMIZATION PROBLEM FORMULATION

The optimization variables in our problem are defined as follows: $\rho_{m,k}(n)$ is the binary assignment variable (BAV), which assigns the $n^{th}$ RB to the backhaul link of the $m^{th}$ RS in the 1st TS and the relay access link to the $k^{th}$ UE in the 2nd TS (in-band relaying), as shown in Fig. 2. $\gamma_k(n)$ and $\beta_k(n)$ are BAVs, which assign the $n^{th}$ RB to the access link to the $k^{th}$ UE in the 1st and 2nd TSs, respectively. The
power values transmitted by the eNB on the $n^{th}$ RB in the 1st and 2nd TSs are $P_{1}^{\text{eNB}}(n)$ and $P_{2}^{\text{eNB}}(n)$, respectively. Finally, the power transmitted by the $m^{th}$ RS on the $n^{th}$ RB in the 2nd TS is $p_{m}^{\text{RS}}(n)$. The achievable rate of the $k^{th}$ UE relayed by the $m^{th}$ RS is thus given by [14]:

$$R_{m,k}^{\text{CE}} = \sum_{n=1}^{N} p_{m,k}(n) \times R_{m,k}^{\text{CE}}(n)$$

(1)

where

$$R_{m,k}^{\text{CE}}(n) = \frac{1}{2} \min \left\{ \frac{C \left( \text{SNR}_{eNB-RS_m}(n) \right)}{C \left( \text{SNR}_{RS_m-UE_k}(n) \right)} \right\}$$

(2)

$$\text{SNR}_{eNB-RS_m}(n) = \frac{p_{1}^{\text{eNB}}(n)h_{m}(n)}{N_0W}$$

(3)

$$\text{SNR}_{RS_m-UE_k}(n) = \frac{p_{m}^{\text{RS}}(n)h_{m,k}(n)}{N_0W + p_{2}^{\text{RS}}(n)h_{k}(n)}$$

(4)

and $C(x) \triangleq \log_2(1 + x)$ is Shannon capacity. Eqs. (3) and (4) represent the SNRs and SINRs of the received signal at $RS_m$ and $UE_k$ in the 1st and 2nd TSs, respectively. In these equations, $h_{m}(n)$ represents the channel gain coefficient between the eNB and the $x^{th}$ node ($RS_m$ or $UE_k$) on the $n^{th}$ RB, and $h_{m,k}(n)$ represents the channel gain coefficient between the $m^{th}$ RS and the $k^{th}$ UE on the $n^{th}$ RB. Both small scale and large scale fading effects are embedded in the channel gain coefficients. As for the achievable rate at the $k^{th}$ CC UE (served only by the eNB), it is given by:

$$R_{k}^{\text{CC}} = \frac{1}{2} \sum_{n=1}^{N} \gamma_{k}(n) \times R_{k}^{\text{CC},1}(n) + \frac{1}{2} \sum_{n=1}^{N} \beta_{k}(n) \times R_{k}^{\text{CC},2}(n)$$

(5)

where

$$R_{k}^{\text{CC},1}(n) = C \left( \text{SNR}_{eNB-UE_k}(n) \right)$$

(6)

and $\text{SNR}_{eNB-UE_k}(n) = \frac{p_{eNB}^{\text{eNB}}(n)h_{k}(n)}{N_0W}$

(7)

represent the capacity and received SINR, respectively, at the $k^{th}$ CC UE on the $n^{th}$ RB in the 1st TS. Similarly,

$$R_{k}^{\text{CC},2}(n) = C \left( \text{SNR}_{RS_m-UE_k}(n) \right)$$

(8)

$$\text{SNR}_{RS_m-UE_k}(n) = \frac{p_{RS}^{\text{RS}}(n)h_{m,k}(n)}{N_0W + \sum_{m \in \mathcal{M}} p_{m}^{\text{RS}}(n)h_{m,k}(n)}$$

(9)

represent the capacity and received SINR, respectively, at the $k^{th}$ CC UE on the $n^{th}$ RB in the 2nd TS.

We can then formulate the resource allocation problem including RB assignment and power allocation as an optimization problem for the worst UE capacity as follows:

$$\max_{\rho, \gamma, \beta} \min_{k \in \mathcal{K}^{CE}} \left\{ \min_{m \in \mathcal{M}} R_{m,k}^{\text{CE}}, \min_{k \in \mathcal{K}^{CC}} R_{k}^{\text{CC}} \right\}$$

(10a)

subject to

$$\sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}^{CE}} p_{m,k}(n) + \sum_{k \in \mathcal{K}^{CC}} \gamma_{k}(n) \leq 1, \quad \forall \ n \in \mathcal{N},$$

(10b)

$$\sum_{k \in \mathcal{K}^{CC}} \beta_{k}(n) \leq 1, \quad \forall \ n \in \mathcal{N},$$

(10c)

$$RB_{\text{CE}}^{\text{min}} \leq \sum_{n=1}^{N} \rho_{m,k}(n) \leq RB_{\text{CE}}^{\text{max}}, \quad \forall \ k \in \mathcal{K}^{CE},$$

(10d)

$$RB_{\text{CC}}^{\text{min}} \leq \sum_{n=1}^{N} \beta_{k}(n) \leq RB_{\text{CC}}^{\text{max}}, \quad \forall \ k \in \mathcal{K}^{CC},$$

(10e)
The objective of the problem is to get the optimal RB assignment and power allocation for the cell of interest to maximize the rate of the worst user (which will be called the bottleneck capacity in the rest of the paper). This criterion was chosen to prevent the multiuser system from falling in an outage state, as discussed before, by focusing on the worst user. Constraint (10b) means that each RB is assigned to only 1 relayed data stream (eNB-RS\_m-UE\_k) in the 2 TSs, or assigned to a direct link to the CC UE in the 1st TS. Constraint (10c) ensures orthogonality between different direct links in the 2nd TS. Some UEs may end up obtaining a huge number of RBs or no RBs at all according to their channel conditions with the eNB and RSs. Accordingly, constraints (10d) and (10e) are added to prevent such scenario. Constraints (10f) and (10g) represent the power budget of the eNB and RSs, respectively. Constraints (10h)-(10l) are added to ensure that power is not allocated to unused RBs that are not assigned to any node. Finally, constraints (10k) and (10l) are the non-negativity and binary constraints put on power variables and BAVs, respectively. This problem can be simplified by introducing the variables R\_CE\_min, R\_CC\_min, and R\_min that correspond to the bottleneck CE UE capacity, the bottleneck CC UE capacity, and the bottleneck UE capacity, respectively. So, the simplified RA problem can be written as:

\[
\begin{align*}
\max_{\rho, \beta, \rho_{\min}, \beta_{\min}, R_{\min}} & \quad R_{\min} \\
\text{subject to} & \quad \text{Constraints (10b)–(10l)}
\end{align*}
\]  

Plugging Eq. (2) and Eq. (5) into the optimization problem, we get the following formulation:

\[
\begin{align*}
\max_{\rho, \beta, \rho_{\min}, \beta_{\min}, R_{\min}} & \quad R_{\min} \\
\text{subject to} & \quad \text{Constraints (10b)–(10l)}
\end{align*}
\]
IV. SOLUTION APPROACH

A well-known solution approach for such a complex RA problem is the coordinate ascent (CA) approach, which was used in similar works [7]. The approach is to develop an iterative algorithm, in which each iteration consists of 2 stages: a stage that optimizes on the RB assignment vectors and the other one optimizes on the power vectors. A typical initial point is to assume equal power allocation, where the total transmit power of eNB is divided equally among the $N$ RBs and the total transmit power of each RS is divided among its legacy CE UEs. Then, optimize on the RB assignment vectors given equal power allocation. Using the RB assignment of the $1^{st}$ stage, the $2^{nd}$ stage will optimize on the power vector using the DC programming method. The algorithm will then iterate until the changes in $R_{\text{min}}$ fall below a certain prescribed value $\epsilon$. To speed the overall algorithm convergence, each PA step uses the optimal point of the last PA step as an initial point. This algorithm is not guaranteed to reach a global optimum, but it should converge to a local optimum, which is still a satisfying solution as will be shown in Section VI. The two sub problems are demonstrated in the following subsections.

A. The RBA sub problem

In the $1^{st}$ stage, the power vector is fixed. Accordingly, $R_{\text{min}}^{\text{CE}}(n)$ will be fixed w.r.t. the RB assignment vectors. The optimization problem will hence be formulated as a Mixed Integer Linear Program (MILP):

$$\max_{\rho_{m,\text{min}}, R_{\text{min}}^{\text{CE}}, \rho_{m,\text{CE}}} R_{\text{min}}$$
subject to

$$(10b)-(10c), (10f), (12c)-(12f)$$

The above problem can be solved by any MILP software.

B. The PA Sub problem

In the $2^{nd}$ stage, the RB assignment variables are fixed and the optimization problem will be formulated as:

$$\max_{\rho_{m,\text{min}}, R_{\text{min}}^{\text{CE}}, \rho_{m,\text{CE}}} R_{\text{min}}$$
subject to

Constraints $(10f)-(10i), (12c)-(11f)$

Problem (13) is a nonconvex problem because of the interference terms in $\text{SINR}_{RS_{m}-UE_k}$ in constraint $(12d)$ and $\text{SINR}_{eNB-UE_k,2}$ in constraint $(12e)$. These constraints can be reformulated as a difference between two concave functions as follows:

$$C(\text{SINR}_{RS_{m}-UE_k}(n)) = \log_2 \left( 1 + \frac{p^{\text{en}}(n)h_{2}(n)}{N_0W + p^{\text{en}}(n)h_{m',k}(n)} \right)$$

$$u_{m',k,n}(p) = \log_2 \left( N_0W + p_{m'}^{\text{en}}(n)h_{m',k}(n) + p^{\text{en}}(n)h_{2}(n) \right)$$

$$v_{m',k,n}(p) = \log_2 \left( N_0W + p_{m'}^{\text{en}}(n)h_{m',k}(n) \right)$$

are concave functions in the power vector. In Eqs. (14) and (15), $m^*$ represents the index of the RS that reuses the $n^{th}$ RB, where $m^*$ can be identified by knowing the RB assignment variables. Similarly,

$$C(\text{SINR}_{RS_{m}-UE_k}(n)) = \log_2 \left( 1 + \frac{p_{m'}^{\text{en}}(n)h_{m,k}(n)}{N_0W + p^{\text{en}}(n)h_{2}(n)} \right)$$

$$u_{m',k,n}(p) = \log_2 \left( N_0W + p_{m'}^{\text{en}}(n)h_{2}(n) + p^{\text{en}}(n)h_{m,k}(n) \right)$$

$$v_{m',k,n}(p) = \log_2 \left( N_0W + p_{m'}^{\text{en}}(n)h_{2}(n) \right)$$

are concave functions in the power vector. The non-concave parts in Eqs. (14) and (15) can be linearized using the $1^{st}$ order Taylor approximation around $p^{(i)}$ as shown:

$$C(\text{SINR}_{RS_{m'-UE_k,2}}) = u_{m',k,n}(p) - v_{m',k,n}(p^{(i)}) - \frac{d u_{m',k,n}}{d p^{(i)}} \times (p - p^{(i)})$$

$$C(\text{SINR}_{RS_{m}-UE_k}(n)) = f_{m,k,n}(p) - g_{m,k}(p^{(i)}) - \frac{d f_{m,k,n}}{d p^{(i)}} \times (p - p^{(i)})$$
where \( \mathbf{p}^{(i)} \) is the power vector obtained in the previous iteration. The problem now becomes an affine maximization problem under concave and affine constraints set. It can be solved using regular convex optimization solvers. The overall algorithm will be referred to as Optimized PA + Optimized RBA (OPA+ORBA).

V. ANOTHER SOLUTION APPROACH

In this section, another solution approach, which is based on similar works, will be presented to compare its performance with our approach. This approach is based on dividing the RA problem into two sub problems which are elaborated in the following subsections.

A. Low Complexity Heuristic for RBA Problem

In [6], the authors proposed a low complexity heuristic for allocating subchannels to relays in a simple relay-enhanced network. We added some modifications to this algorithm to make it suitable for our system model. Channel gain averaged on all RBs is used as a metric for comparing UEs. This is because equal power allocation is assumed initially, and intra-cell interference is neglected. The algorithm first sorts CE UEs according to their average channel gains and allocates RBs to them according to their new order. Then, it sorts all UEs according to their average channel gains, and assigns the remaining RBs to the remaining UEs (in the 1st and 2nd TSs for CE UEs relayed transmissions and in the 1st TS for CC UEs) until the RBs are exhausted. After that, CC UEs that were not assigned to any RBs in the 1st TS are sorted as previous, and then they are assigned to RBs according to their new order.

B. Solving the PA problem using Iterative Multi-Level WF Algorithm (IMLWF)

In our problem, there are \( M + 2 \) transmitting nodes, these nodes are: 1) eNB that transmits to RSs, and CC UEs in the 1st TS, 2) eNB that transmits to CC UEs in the 2nd TS, and 3) RSs that transmit to their legacy CE UEs in the 2nd TS. The main objective is to find the transmitting power of all nodes to maximize the bottleneck capacity as in Eq. (12a). We will approximate this objective to finding the transmitting power of each node to maximize bottleneck capacity among the legacy users only. In this way, we can divide the main PA problem into multiple PA sub problems of the form:

\[
\max_{P_{k,l}} R_{\text{min}}^{(k,l)} \quad \text{subject to} \quad \sum_{i=1}^{L_k} \log_2(1 + P_{k,l} \lambda_{k,i}) \geq R_{\text{min}} \forall k, l \quad (28a)
\]

\[
\sum_{k=1}^{K} \sum_{l=1}^{L_k} P_{k,l} \leq P_{\text{max}} \quad (28b)
\]

\[
R_{\text{min}} \geq 0, P_{k,l} \geq 0, \forall k \in \{1, ..., K\}, l \in \{1, ..., L_k\}. \quad (28d)
\]

In this sub problem, the \( k^{\text{th}} \) user is assigned to \( L_k \) RBs, which are determined by \( \rho_{mk}(n), \gamma_k(n), \beta_k(n) \) calculated in the RB assignment heuristic. \( P_{k,l} \) is the power assigned to the \( k^{\text{th}} \) user on the \( l^{\text{th}} \) assigned RB. \( \lambda_{k,i} \) is the Channel-to-Interference-plus-Noise Ratio (CINR), where the interference power is considered as a fixed constant known from the last iteration. The Lagrangian of this problem can be written as:

\[
L = R_{\text{min}} + \sum_{k=1}^{K} a_k \left( R_{\text{min}} - \sum_{i=1}^{L_k} \log_2(1 + P_{k,l} \lambda_{k,i}) \right) + \frac{\eta}{\ln 2} \sum_{k=1}^{K} \sum_{l=1}^{L_k} P_{k,l} - P_{\text{max}} \quad (29)
\]

Differentiating w.r.t. \( P_{k,l} \), and equating to zero, we get the following equation after some manipulations:

\[
P_{k,l} = \left( \mu_k - \lambda_{k,i}^{-1} \right)^+ \quad (30)
\]

where \( \mu_k = \frac{a_k}{\ln 2} (x)^+ \) is the max value of \( x \). This solution can be interpreted as a Multi-Level Water Filling (MLWF) solution, where the \( k^{\text{th}} \) user has its own water level (\( \mu_k \)). A practical algorithm for calculating these water levels is proposed in [15]. This algorithm tends to strictly equalize all users’ rates, which is the long-term behaviour of max-min fair RA problems. We apply MLWF algorithm on eNB and each RS separately in 2nd TSs to allocate power to their legacy users, and we apply the regular water filling solution to eNB in 1st TS to assign power to CC UEs and RSs. To account for the intra-cell interference in 2nd TS, we will use the Iterative Water-Filling (IWF) concept proposed in [16]. Each node (eNB or RS) will allocate its power assuming interference of the other node as noise. Then, the interference power is updated and the algorithm iterates until it reaches a Nash equilibrium point. The overall algorithm will be referred to as IMLWF+Heuristic RBA (IMLWF+HRBA).
VI. PERFORMANCE EVALUATION

In this section, we show performance of our algorithm compared to the IMLWF+HRBA algorithm using a MATLAB system-level simulator built for this problem. Uniform user distribution is assumed all over the cell. We use 10 different random samples, where the problem is solved in each sample for 10 consecutive Transmission Time Intervals (TTIs). Channel gains of all links are calculated using Winner II channel model [17]. The macro access links, relay access links, and backhaul links are modeled as typical urban macro-cell (C2), typical urban micro-cell (B1), and feeder link (B5f), respectively. Other simulation parameters are shown in Table 1.

The RB assignment problem is solved using CPLEX 12.5 [18], the PA optimization problem is modelled using CVX [19] and solved using MOSEK solver. In the overall algorithm, each PA sub problem is solved using the DC technique.

<table>
<thead>
<tr>
<th>Table 1: Simulation Parameters</th>
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<tbody>
<tr>
<td>Parameter</td>
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<tr>
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<tr>
<td>Cell Radius (R)</td>
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<tr>
<td>Number of RBs (N)</td>
</tr>
<tr>
<td>Number of UEs (K)</td>
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<tr>
<td>Number of RSs (M)</td>
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<tr>
<td>Distance of RSs from the center (Rrs)</td>
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<tr>
<td>Threshold average channel gain (( A_{th} ))</td>
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<tr>
<td>eNB transmit power (( P_{\text{max}}^{\text{eNB}} ))</td>
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<tr>
<td>RS transmit power (( P_{\text{max}}^{\text{RS}} ))</td>
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<tr>
<td>Thermal noise (( N_0 ))</td>
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The convergence of the overall algorithm for an arbitrary sample and arbitrary TTI is shown in Fig. 3, where the number of CC and CE UEs are 39 and 21, respectively. It can be inferred that the algorithm tends to increase the bottleneck capacity (CE or CC) steadily at the beginning, then the marginal increase become very small. Practically, we stop the algorithm after a lower number of iterations by carefully choosing the tolerance value \( \varepsilon \). Moreover, it can be noted that the algorithm prevents the value of bottleneck CC capacity from falling below the value of bottleneck CE capacity. This is because increasing the rate of a CC UE is much easier than increasing the rate of a CE UE due to the higher transmission power of eNB compared to RSs. On the other hand, the IMLWF+HRBA algorithm is simpler as it consists of a single RB assignment step and single PA step that uses IMLWF algorithm. For comparison, another simplified technique is used which is similar to OPA+ORBA algorithm; however, the optimal PA solution is replaced by an Equal Power Allocation (EPA) in each iteration. This technique is denoted by EPA+ORBA.

Figs. 4 and 5 show the CDF of the bottleneck capacities and all capacities for the 3 algorithms using 10 random samples and 10 TTI/sample. It is shown that OPA+ORBA algorithm outperforms the other 2 algorithms in both the bottleneck capacity and all user capacities. The values of 10%tile capacity for the IMLWF+HRBA, EPA+ORBA, and OPA+ORBA algorithms are 0.28 bps/Hz, 0.77 bps/Hz, and 1.86 bps/Hz, respectively. In other words, our algorithm OPA+ORBA obtains a capacity gain factor of 6.6 and 2.4 compared to IMLWF+HRBA and EPA+ORBA algorithms, respectively.
To quantify the fairness among users, we use the Jain’s index [5], a well-known fairness index. Fig. 6 shows the CDF of Jain’s index for the 3 algorithms. It is shown that OPA+ORBA algorithm achieves the best fairness, which is expected because of its max-min fair criterion. It is also noted that EPA+ORBA algorithm achieves the least fairness because the EPA is not a fair way of allocating power to UEs that have different channel gains.

VII. CONCLUSIONS

In this paper, we proposed a new technique for solving the complex RA problem in OFDMA REC's that employ spatial reuse. This technique allocates the power and RBs in a max-min fair sense, which is important for multiuser systems. Comparison with other techniques showed that the proposed algorithm achieves better solutions and more fairness; however, it is more complex than the other techniques. As a future work, this technique needs to be also applied and tested in a multi-cell scenario.

REFERENCES


